Mining Changes and Connections using Rough Set Theory

Ken Kaneiwa
National Institute of Information and Communications Technology
3-5 Hikaridai, Seika, Soraku, Kyoto 619-0289, Japan
kaneiwa@nict.go.jp

Abstract

Mining data changes and connections from information systems (or databases) is made difficult by the different data behaviors and relationships across multiple data sets. When making a decision, such a dynamic and integrated knowledge base can be used to set useful rules (e.g., causality) that differ from the statistical associations in a single resource. In this paper, using techniques based on the rough set theory, we propose a change and connection mining algorithm for discovering a time delay between the quantitative changes in the data of two temporal information systems and for generating the association rules of changes from their connected decision table. We establish evaluation criteria for the connectedness of two temporal information systems with varying time delays by calculating weight-based accuracy and coverage of the association rules of changes, adjusted by a fuzzy membership function.

keywords: rough set theory, fuzzy theory, change and connection mining, causality

1 Introduction

As a tool for data analysis, data mining algorithms [3, 7] enable us to discover useful patterns and rules from information systems (or databases). In the algorithms, association rules are generated from the frequent attributes (or itemsets) in an information system. In addition, numerous techniques for extracting association rules (known as decision rules) have been proposed in the field of rough set theory [19, 15, 21, 16]. Unlike conventional algorithms for mining statistical associations, the rough set approach provides an algorithm for logically formulating association rules in a decision table. More specifically, minimal and consistent association rules are computed using the

[1] This paper is an extended version of [13].

lower approximation of a rough set by checking for any logical combination of condition and decision attributes.

More importantly, the integration of multiple information systems is carried out to create useful data that a single information system does not yield. In the environment of computer systems, sources of data are distributed across multiple sites, contexts, and domains (e.g., data on the web). Even in a local site, (e.g., in computer systems of enterprises), multiple information systems are often separated and incompatible with each other when independently designed by users during a long period. Therefore, there is an urgent requirement to integrate and analyze distributed data for discovering valuable patterns and rules across multiple information systems. The mining of association rules from multiple information systems has to be realized in a highly sophisticated manner, as it involves analyzing and integrating various data in different contexts of the systems. Existing data mining algorithms [5, 12, 25] effectively and statistically integrate multiple data resources; however, they do not attempt to establish data behaviors and relationships across multiple data sets.

To enable integration and discovery, time and space stamps should be used as references for examining changes and connections in different data sets. The references can be used to indicate a time delay between the time-stamped data of two distributed information systems. The *quantitative* changes in the temporal data of such information systems can be interpreted as events; therefore, a time delayed connection implies that one quantitative change causes another. From this perspective, a candidate causality is analyzed and obtained by connecting the quantitative changes in the distributed data.

Causality [20, 10] derived from changes and connections is valuable knowledge obtained from the integration of multiple information systems. This is because changing the values of data in one context may affect the values of data in another context. Therefore, over and above the statistical associations in a single information system, such causal knowledge has an advantage in that each of the conditions implies a different effect. However, there are few approaches to mining the changes and connections in data across multiple information systems. As discussed in [11, 23], most knowledge discovery algorithms capture only statistical associations that are substantially different from causality.

In this paper, we propose a change and connection mining algorithm based on the notions of attribute reduction and minimal rule generation in the rough set theory. We assume that attribute data in information systems are associated with time stamps. For the purpose of mining, we formalize the quantitative changes that are estimated using different operators; these results are used to derive the association rules of changes by slidingly connecting two *temporal* information systems for varying time delays. In order to measure the changes (including *fast changing events*), we propose *weight-based* accuracy and coverage of the association rules with respect to the indiscernibility of changes. These evaluations identify the connectedness of the two temporal information systems for each *time delay.*

Our algorithm for mining changes and connections has the following interesting features.

- **Minimal rules:** The minimal rule generation in the rough set theory enables us to obtain minimal association rules using attribute reduction. Minimal causes provide better explanations of effects, as employed in abductive reasoning [22].

- **Consistent rules:** The discernibility of decision classes in the rough set theory is used to derive consistent rules, i.e., logically, inconsistent rules $\varphi \to \alpha$ and $\varphi \to \neg\alpha$ are excluded. As a result, the consistency is suitable for establishing the connectedness of distributed data due to the small conflict among the consistent rules.

- **Connections between quantitative changes:** In order to discover a strong connection between information systems, we evaluate the association rules of changes that are generated from the quantitative changes in estimating the values of data.

- **Time delays:** Mining of changes and connections is realized by considering different time delays between the data of two temporal information systems. This is temporally consistent because of the adherence to the assumption in the causal theory [24], i.e., "if A causes B, then A occurs earlier than B."

These features follow the three classical conditions under which A causes B (as defined in [11]): (i) statistical associations between the values of A and B, (ii) the direction of causality, and (iii) no common causes of A and B.

This paper is arranged as follows. Section 2 briefly recalls the basic notions of rough sets. In Section 3, we describe a connection method of two temporal information systems for various time delays. As part of the method, we propose the notions of quantitative changes in attribute values and the association rules of changes. Then, we establish the connectedness of two temporal information systems for each time delay that is evaluated by weight-based accuracy and coverage. In Section 4, we present our algorithm for mining changes and connections from two temporal information systems. The experimental results are reported in Section 5. Finally, we discuss the related work to our approach in Section 6 and conclude this paper in Section 7.

2 Rough Sets

An attribute a is a mapping $a\colon U \to V_a$ where U is a non-empty finite set of objects (called the universe) and V_a is the value set of a. An information system is a pair $T = (U, A)$ of the universe U and a non-empty finite set A of attributes. Let B be a subset of A. The B-indiscernibility relation is defined by an equivalence relation I_B on U such that $I_B = \{(x, y) \in U^2 \mid \forall a \in B.a(x) = a(y)\}$. The equivalence class of I_B for each object x ($\in U$) is denoted by $[x]_B$. Let X be a subset of U. We define the lower and upper approximations of X by $\underline{B}(X) = \{x \in U \mid [x]_B \subseteq X\}$ and $\overline{B}(X) = \{x \in U \mid [x]_B \cap X \neq \emptyset\}$. A subset B of A is a reduct of T if $I_B = I_A$ and there is no subset B' of B with $I_{B'} = I_A$ (i.e., B is a minimal set of attributes without losing discernibility).

A decision table is an information system $T = (U, A \cup \{d\})$ such that each $a \in A$ is a condition attribute and $d \notin A$ is a decision attribute. Let V_d be the value set $\{d_1, \ldots, d_u\}$ of the decision attribute d. For each value $d_i \in V_d$, we obtain a decision class $U_i = \{x \in U \mid d(x) = d_i\}$ where $U = U_1 \cup \cdots \cup U_{|V_d|}$ (i.e., $u = |V_d|$) and for every $x, y \in U_i$, $d(x) = d(y)$. The B-positive region of d is defined by $P_B(d) = \underline{B}(U_1) \cup \cdots \cup \underline{B}(U_{|V_d|})$. A subset B of A is a relative reduct of T if $P_B(d) = P_A(d)$ and there is no subset B' of B with $P_{B'}(d) = P_A(d)$.

We define a formula $(a_1 = v_1) \wedge \cdots \wedge (a_n = v_n)$ in T (denoting the condition of a rule) where $a_j \in A$ and $v_j \in V_{a_j}$ $(1 \leq j \leq n)$. The semantics of the formula in T is defined by $[\![(a_1 = v_1) \wedge \cdots \wedge (a_n = v_n)]\!]_T = \{x \in U \mid a_1(x) = v_1, \ldots, a_n(x) = v_n\}$. Let φ be a formula $(a_1 = v_1) \wedge \cdots \wedge (a_n = v_n)$ in T. A decision rule for T is of the form $\varphi \to (d = d_i)$, and it is true if $[\![\varphi]\!]_T \subseteq [\![(d = d_i)]\!]_T (= U_i)$. The accuracy and coverage of a decision rule r of the form $\varphi \to (d = d_i)$ are respectively defined as follows.

$$accuracy(T, r, U_i) = \frac{|U_i \cap [\![\varphi]\!]_T|}{|[\![\varphi]\!]_T|}$$
$$coverage(T, r, U_i) = \frac{|U_i \cap [\![\varphi]\!]_T|}{|U_i|}$$

In the evaluations, $|U_i|$ is the number of objects in a decision class U_i and $|[\![\varphi]\!]_T|$ is the number of objects in the universe $U = U_1 \cup \cdots \cup U_{|V_d|}$ that satisfy condition φ of rule r. Therefore, $|U_i \cap [\![\varphi]\!]_T|$ is the number of objects satisfying the condition φ restricted to a decision class U_i.

3 Changes and Connections in Data

In this section, we present a connection method for two temporal information systems with various time delays. We establish the quantitative changes in the attribute values of temporal information systems and formulate the association rules of changes in a connected decision table. For each of the time delays, the connectedness of two temporal information systems is evaluated by our proposed weight-based accuracy and coverage.

3.1 Connecting Information Systems

A *temporal* information system is an information system $T = (U_{time}, A)$ where the objects x of U_{time} denote time stamps (e.g., dates and weeks) and for each attribute $a \in A$, $a(x)$ maps the value of a at time stamp x.

Each temporal information system $T = (U_{time}, A)$ is normalized such that all the elements of U_{time} are replaced by natural numbers. Let $Nat_{i,j}$ denote a finite set of natural numbers such that $\{x \in Nat \mid i \leq x \leq j\}$. Then, we define the normalization of a temporal information system $T = (U_{time}, A)$ by $N(T) = (Nat_{i,j}, A')$ with a bijection $n\colon U_{time} \rightarrow Nat_{i,j}$ such that for every $x, y \in U_{time}$, $x < y \Leftrightarrow n(x) < n(y)$, $|U_{time}| = |Nat_{i,j}|$, and $A' = \{a' \mid a \in A \;\&\; \forall x \in U_{time}.a'(n(x)) = a(x)\}$. For example, $T_1 = (\{2001\text{-}1\text{-}1, 2001\text{-}1\text{-}2, 2001\text{-}1\text{-}3\}, A_1)$ is normalized by the natural number set $Nat_{1,3} = \{1, 2, 3\}$, i.e., $N(T_1) = (Nat_{1,3}, A'_1)$ where $n(2001\text{-}1\text{-}1) = 1$, $n(2001\text{-}1\text{-}2) = 2$, and $n(2001\text{-}1\text{-}3) = 3$.

Lemma 1 *Every temporal information system is translated into a normalized temporal information system.*

In the remainder of this paper, we will focus on normalized temporal information systems because of this lemma.

We define a connection of two normalized temporal information systems with a time delay by matching and arranging time stamps.

Definition 1 (Connections with a Time Delay) *Let $T_1 = (Nat_{i,j}, A_1)$ and $T_2 = (Nat_{h,m}, A_2)$ be two normalized temporal information systems such that T_1 and T_2 have no common attributes ($A_1 \cap A_2 = \emptyset$). The connection $con(T_1, T_2, \Delta)$ of T_1 and T_2 with a time delay Δ ($\in Nat$) is defined as an information system $T = (U_{time}, A)$ such that*

- $U_{time} = Nat_{i,j} \cap Nat_{h',m-\Delta}$ *and*
- $A = \{a \restriction U_{time} \mid a \in A_1\} \cup \{b' \mid b \in A_2 \;\&\; \forall x \in U_{time}.b'(x) = b(x - \Delta)\}$

where $a \restriction U_{time}$ denotes the attribute a restricted to domain U_{time}, and $h' = i$ if $h - \Delta < i$, otherwise, $h' = h - \Delta$.

In Figure 1, the tables on the left-hand side present two normalized temporal information systems $T_1 = (Nat_{1,7}, A_1)$ and $T_2 = (Nat_{2,6}, A_2)$, and the table on the right-hand side shows the connection $con(T_1, T_2, 1)$ of T_1 and T_2. The time stamps of attributes b_1 and b_2 in T_2 are decreased by the time delay $\Delta = 1$. For example, time stamp 3 of values 2 and 1 of b_1 and b_2 in T_2 is changed

time	a_1	a_2
1	3	1
2	3	1
3	3	2
4	5	3
5	16	4
6	3	4
7	1	2

&

time	b_1	b_2
2	2	1
3	2	1
4	1	3
5	3	4
6	5	5

$\Longrightarrow$

time	a_1	a_2	b_1	b_2
1	3	1	2	1
2	3	1	2	1
3	3	2	1	3
4	5	3	3	4
5	16	4	5	5

Figure 1: The connection $con(T_1, T_2, 1)$ of $T_1 = (Nat_{1,7}, A_1)$ and $T_2 = (Nat_{2,6}, A_2)$ with time delay $\Delta = 1$.

into time stamp 2 in $con(T_1, T_2, 1)$. As a result of the connection, the lowest two rows in T_1 are deleted to adjust for the size of T_2. Furthermore, this connection method can be applied to more than two information systems. For example, the connection $con(con(T_1, T_2, \Delta_1), T_3, \Delta_2)$ is obtained when we integrate three information systems T_1, T_2, and T_3 with time delays Δ_1 and Δ_2.

Let $T_1 = (U_{time}, A_1)$ and $T_2 = (U'_{time}, A_2)$ be two temporal information systems. Their connection builds a decision table $T = con(T_1, T_2, \Delta)$ in the rough set theory if A_2 is a singleton as a decision attribute. If A_2 is not a singleton, then T_2 is reduced to an information system $T_2[b] = (U'_{time}, \{b\})$ for an attribute $b \in A_2$. Alternatively, we can translate from A_2 into a singleton without losing the information. Let $A_2 = \{b_1, \ldots, b_n\}$. Then, the translation is defined such that the attributes $b_1, \ldots, b_n$ are expressed by an n-tuple, i.e., $T_2^f = (U'_{time}, \{f_{(b_1,\ldots,b_n)}\})$ where for every $x \in U'_{time}$, $f_{(b_1,\ldots,b_n)}(x) = (b_1(x), \ldots, b_n(x))$. Therefore, we obtain two decision tables $con(T_1, T_2[b], \Delta)$ and $con(T_1, T_2^f, \Delta)$. If T_1 and T_2 are regarded as cause and effect information systems, then the connection of T_1 and T_2 with a time delay leads to a cause-effect decision table. However, if the data behavior cannot be interpreted in terms of a decision table, a cause-effect decision table cannot be generated. In addition to the varying of time delays, we have to extract data behaviors by estimating the changes in the data values of both cause and effect information systems.

3.2 Changes and Time Delays

In order to classify data behaviors such as increase and decrease, we define the *quantitative changes* in the data values of temporal information systems. Given two temporal information systems T_1 and T_2, their quantitative changes may represent a connection between the candidate data of causes and effects. Through the diversity of time delays, numerous connections of the quantitative changes will be analyzed to determine whether or not changing the values of attributes in an information system T_1 affects the values of attributes in another information system T_2.

Definition 2 (Quantitative Estimation Operators)
Several quantitative estimation operators for the numeric values $a(x)$ of attributes are defined by the following.

$$\textit{(difference)}\ \pi_d(a(x)) = a(x) - a(x-1)$$
$$\textit{(variation rate)}\ \pi_v(a(x)) = \frac{a(x) - a(x-1)}{a(x)}$$
$$\textit{(threshold)}\ \pi_{\geq k}(a(x)) = \begin{cases} 1 & \textit{if}\ \ a(x) \geq k \\ 0 & \textit{otherwise} \end{cases}$$
$$\textit{(variation rate of difference)}\ \pi_{2v}(a(x)) = \pi_v(\pi_d(a(x)))$$

$$\textit{(trend)}\ \pi_{tr(k)}(a(x)) = \frac{a(x-k)+\cdots+a(x)+\cdots+a(x+k)}{2k+1}$$

These operators are (sometimes compositionally) used to estimate the quantitative changes in temporal information systems T as follows.

Definition 3 (Quantitative Changes) *Let $T = (Nat_{i,j}, A)$ be a normalized temporal information system. The quantitative change of T obtained by a quantitative estimation operator $\pi \in \{\pi_d, \pi_v, \pi_{\geq k}, \pi_{\leq k}, \pi_{2v}, \pi_{tr(k)}\}$ with $h, m \in Nat$ is an information system $\pi(T) = (U_{time}, A')$ such that*

- $U_{time} = Nat_{i+h,j-m}$ *with* $i + h \leq j - m$ *and*
- $A' = \{a' \mid a \in A\ \&\ \forall x \in U_{time}.a'(x) = \pi(a(x))\}$

where $h = 1$ and $m = 0$ if $\pi = \pi_d$ or π_v, $h = m = 0$ if $\pi = \pi_{\geq k}$ or $\pi_{\leq k}$, $h = 2$ and $m = 0$ if $\pi = \pi_{2v}$, and $h = k$ and $m = k$ if $\pi = \pi_{tr(k)}$.

The natural numbers h and m indicate that $Nat_{i,j}$ is reduced to $Nat_{i+h,j-m}$ because the number of values obtained by some operators decreases. For example, Figure 2 shows the differences $\pi_d(T_1) = (Nat_{2,7}, A'_1)$ and the results of the high threshold $\pi_{\geq 2.5}(T_2) = (Nat_{2,6}, A'_2)$ estimated from $T_1 = (Nat_{1,7}, A_1)$ and $T_2 = (Nat_{2,6}, A_2)$, respectively.

In the process of data analysis, we are not certain about the temporal relationship between two information systems; therefore, they should be combined exhaustively in a scope of time delays. By increasing the time delay Δ from 0, the quantitative changes $\pi_1(T_1)$ and $\pi_2(T_2[b])$ of two temporal information systems T_1 and $T_2[b]$ are slidingly connected to achieve numerous cause-effect decision tables as follows.

$$con(\pi_1(T_1), \pi_2(T_2[b]), 0), con(\pi_1(T_1), \pi_2(T_2[b]), 1),$$
$$con(\pi_1(T_1), \pi_2(T_2[b]), 2), \ldots, con(\pi_1(T_1), \pi_2(T_2[b]), m)$$

time	a_1	a_2
1	3	1
2	3	1
3	3	2
4	5	3
5	16	4
6	3	4
7	1	2

$\Longrightarrow$

time	a_1	a_2
2	0	0
3	0	+1
4	+2	+1
5	+11	+1
6	-13	0
7	-2	-2

time	b_1	b_2
2	2	1
3	2	1
4	1	3
5	3	4
6	5	5

$\Longrightarrow$

time	b_1	b_2
2	0	0
3	0	0
4	0	1
5	1	1
6	1	1

Figure 2: The quantitative changes $\pi_d(T_1)$ and $\pi_{\geq 2.5}(T_2)$.

Let $T = (U_{time}, A)$ with $A = \{a_1, \ldots, a_n\}$ and let quantitative estimation operators $\pi_1, \ldots, \pi_n$ be applied to each attribute a_j in T. Then, the decomposed and estimated information systems $\pi_1(T[a_1]), \ldots, \pi_n(T[a_n])$ are reconnected by

$$con(\pi_1(T[a_1]), con(\pi_2(T[a_2]), \cdots con(\pi_{n-1}(T[a_{n-1}]), \\ \pi_n(T[a_n]), 0) \cdots, 0), 0)$$

which is simply denoted by $\pi_1(T[a_1]) \circ \cdots \circ \pi_n(T[a_n])$. For example, let π_d be a difference operator, π_v be a variation rate operator, and $\pi_{\geq 2.5}$ be a high threshold operator. Figure 3 shows that the quantitative changes $\pi_d(T_1[a_1]) \circ \pi_v(T_1[a_2])$ and $\pi_{\geq 2.5}(T_2[b_1])$ of T_1 and $T_2[b_1]$ in Figure 1 and Figure 2 are transformed into the connected decision table $con(\pi_d(T_1[a_1]) \circ \pi_v(T_1[a_2]), \pi_{\geq 2.5}(T_2[b_1]), 2)$ with time delay $\Delta = 2$ and the decision attribute $b_1 \in A_2$.

3.3 Association Rules of Changes

We define association rules with respect to quantitative changes, which are employed to evaluate the connectedness of two temporal information systems. Let T_1 and $T_2[b]$ be two temporal information systems and π_1 and π_2 be quantitative estimation operators. A decision rule in the rough set theory is called an *association rule of changes* if it is generated from a cause-effect decision table $con(\pi_1(T_1), \pi_2(T_2[b]), \Delta)$ consisting of the quantitative changes $\pi_1(T_1)$ and $\pi_2(T_2[b])$. It should be noted that the association rules of changes $(a_1 = v_1) \wedge \cdots \wedge (a_n = v_n) \rightarrow (b = v)$ adhere to the natures of

time	a_1	a_2
1	0	0.0
2	0	+1.0
3	+2	+0.5
4	+11	+1/3
5	-13	0.0
6	-2	−0.5

&

time	b_1
2	0
3	0
4	0
5	1
6	1

$\Longrightarrow$

time	a_1	a_2	b_1
1	0	0	0
2	0	+1.0	0
3	+2	+0.5	1
4	+11	+1/3	1

Figure 3: The connected decision table $con(\pi_d(T_1[a_1]) \circ \pi_v(T_1[a_2]), \pi_{\geq 2.5}(T_2[b_1]), 2)$ with time delay $\Delta = 2$.

causality as follows.

(i) The cause-effect decision table is built by the quantitative changes in the data of two temporal information systems T_1 and $T_2[b]$ that correspond to the data behaviors of causes and effects.

(ii) A time delay exists such that the time stamps of the condition attributes $a_1 = v_1, \ldots, a_n = v_n$ in $\pi_1(T_1)$ occur earlier than those of the decision attribute $b = v$ in $\pi_2(T_2[b])$.

For example, the following association rules of changes are generated from the connected decision table $con(\pi_d(T_1[a_1]) \circ \pi_v(T_1[a_2]), \pi_{\geq 2.5}(T_2[b_1]), 2)$ in Figure 3.

$$\text{(difference)} \wedge \text{(variation)} \rightarrow \text{(threshold)} \wedge \text{(time delay)}$$
$$(a_1 = 0) \wedge (a_2 = 0.0) \rightarrow (b_1 = 0) \wedge (\Delta = 2)$$
$$(a_1 = +11) \wedge (a_2 = +1/3) \rightarrow (b_1 = 1) \wedge (\Delta = 2)$$

In these rules, the condition attributes are difference π_d and variation rate π_v and the decision attribute is threshold $\pi_{\geq 2.5}$ with time delay $\Delta = 2$. The first rule implies that if the values of attributes a_1 and a_2 are neither decreased nor increased, then the value of attribute b_1 does not exceed threshold 2.5 in the next two time slots. The second rule means that if the value of attribute a_1 is increased by +11 and the variation rate of the value of attribute a_2 is +1/3, then the value of attribute b_1 exceeds threshold 2.5 in the next two time slots.

3.4 Indiscernibility and Weight

In order to interpret the association rules of changes, we would like to make a distinction between the indiscernibility and weight of quantitative changes by improving the evaluation of decision rules in the rough set theory. In this study, indiscernibility captures the increase and decrease of values, and weight measures the quantity of data behaviors.

Let $T = (U_{time}, A \cup \{d\})$ be a decision table and B be a relative reduct of T. The B-indiscernibility relation of quantitative changes is defined by an equivalence relation I_B^{qc} on U_{time} such that

$$I_B^{qc} = \{(x, y) \in U_{time}^2 \mid \forall a \in B.sign(a(x)) = sign(a(y))\}.$$

The sign function $sign(n)$ is defined by $sign(n) = 1$ if $n > 0$, $sign(n) = -1$ if $n < 0$, and $sign(n) = 0$ if $n = 0$.

By the B-indiscernibility I_B^{qc} of quantitative changes, the semantics of the formula $(a_1 = v_1) \wedge \cdots \wedge (a_n = v_n)$ in T is refined by $[\![(a_1 = v_1) \wedge \cdots \wedge (a_n = v_n)]\!]_T^{qc} = \{x \in U_{time} \mid sign(a_1(x)) = sign(v_1), \ldots, sign(a_n(x)) = sign(v_n)\}$. Let $\{s_1, \ldots, s_u\}$ denote $sign(V_d) = \{sign(d_j) \mid d_j \in V_d\}$. For each value s_i of $sign(V_d)$ of the decision attribute d, we define a decision class on quantitative changes $U_i = \{x \in U \mid sign(d(x)) = s_i\}$ where $U = U_1 \cup \cdots \cup U_{|sign(V_d)|}$ (i.e., $u = |sign(V_d)|$) and for every $x, y \in U_i$, $sign(d(x)) = sign(d(y))$.

We consider scanning not only quantitative changes but also those events that drastically change attribute values (which we refer to as *fast changing events*). As a measuring method, the weight $w(x)$ of changes for each time stamp $x \in U_{time}$ is defined by the following.

$$w(x) = \sum_{a \in A \cup \{d\}} ||a(x)||$$

where $||\ ||\colon R \to R$ is the absolute value function such that $||n|| = n$ if $n \geq 0$, otherwise, $||n|| = -n$. For example, the connected decision table in Figure 3 contains fast changing events because $a_1(4) = +11$ is an intensively higher value than the other values. Therefore, we obtain high weight $w(4) = 12.33\cdots$ for time stamp 4 but low weight $w(1) = 0$ for time stamp 1. Weights $w(1)$ and $w(4)$ can be used to respectively evaluate the two association rules $(a_1 = 0) \wedge (a_2 = 0.0) \to (b_1 = 0)$ and $(a_1 = +11) \wedge (a_2 = +1/3) \to (b_1 = 1)$ (shown in Section 3.3). The second association rule importantly includes a fast changing event because of high weight $w(4)$.

Definition 4 (Weight-Based Accuracy and Coverage on I_B^{qc}) *Let U_i be a decision class on quantitative changes and let $x \in U_i$. The weight-based accuracy $w\_accuracy$ and weight-based coverage $w\_coverage$ of a decision rule r of the form $\varphi \to (d = d(x))$ with $d(x) \in V_d$ are defined as follows.*

$$w\_accuracy(T, r, U_i) = \frac{w(U_i \cap [\![\varphi]\!]_T^{qc})}{w([\![\varphi]\!]_T^{qc})}$$

$$w\_coverage(T, r, U_i) = \frac{w(U_i \cap [\![\varphi]\!]_T^{qc})}{w(U_i)}$$

where $w(X) = \sum_{x \in X} w(x)$.

The accuracy and coverage unify similar data behaviors on indiscernibility and measure the quantity of data behaviors on the weight.

3.5 Connectedness

The *connectedness* of two temporal information systems is significantly evaluated by weight-based accuracy and coverage. First, we define a consistency evaluation that finds a most consistent association rule for each decision class. Second, we estimate the total value of the maximum consistency evaluations for all the decision classes.

The consistency evaluation $eval(T, r, U_i)$ is defined by using the weight-based coverage and accuracy of a decision rule r with a fuzzy membership function μ_S as follows.

$$eval(T, r, U_i) = w\_coverage(T, r, U_i) \times \mu_S(w\_accuracy(T, r, U_i))$$

The fuzzy membership function $\mu_S \colon [0, 1] \to [0, 1]$ (similar to the S fuzzy set [6]) is defined by the following.

$$\mu_S(v) = \begin{cases} 0 & \text{if } 0 \leq v \leq \frac{1}{2} \\ 8(v - \frac{1}{2})^2 & \text{if } \frac{1}{2} < v \leq \frac{3}{4} \\ 1 - 8(v - 1)^2 & \text{if } \frac{3}{4} < v \leq 1 \end{cases}$$

By applying the membership function to each accuracy value, we determine whether or not each association rule is suitable for connecting two temporal information systems. In other words, association rules with low accuracy values should be eliminated even if their coverage values are high. This is because such rules are *inconsistent* with some other rules; however, we intend to find *consistent* rules beyond statistical associations. The reason why we use the fuzzy membership function is to emphasize the accuracy values of (in)consistent rules.

For example, consider the following association rules r_1 and r_2 in a connected decision table $T = (U, A \cup \{d\})$.

$$r_1 \colon (a_1 = +1) \wedge (a_2 = -1) \to (d = +1)$$
$$r_2 \colon (a_1 = +1) \wedge (a_2 = -1) \to (d = -1)$$

These association rules have identical conditions; however, their decisions contradict each other. Let $U_1 = \{x \in U \mid d(x) = +1\}$ and $U_2 = \{x \in U \mid d(x) = -1\}$. If we have $w\_accuracy(T, r_1, U_1) = 0.45$ and $w\_accuracy(T, r_2, U_2) = 0.55$, then these rules are inconsistent with each other. Therefore, the membership function results in $\mu_S(0.45) = 0$ and $\mu_S(0.55) = 0.02$, and therefore, the consistency evaluations $eval(T, r_1, U_1) = coverage(T, r_1, U_1) \times 0.02$ and $eval(T, r_2, U_2) = coverage(T, r_2, U_2) \times 0$ return very low values. In contrast, if we have the other values $w\_accuracy(T, r_1, U_1) = 0.89$ and $w\_accuracy(T, r_2, U_2) = 0.1$, then the first rule should be highly evaluated due to the low conflict between r_1 and r_2. In this case, the membership function yields $\mu_S(0.89) = 0.9032$ and $\mu_S(0.1) = 0$. Thus, the accuracy value of the first association rule will affect the consistency evaluation by calculating $eval(T, r_1, U_1) = coverage(T, r_1, U_1) \times 0.9032$.

Let $\mathcal{R}$ be the family of subsets B of A in a decision table $T = (U_{time}, A \cup \{d\})$ such that B is a relative reduct of T. For each reduct $B \in \mathcal{R}$, we can obtain the set R_B of minimal association rules for every decision class U_i, i.e., each reduct B provides a minimal set of condition attributes in A. The maximum consistency evaluation in the set R_B is defined by $max\_eval(T, R_B, U_i) = eval(T, r, U_i)$ if $r \in R_B$ and for every rule $r' \in R_B$, $eval(T, r, U_i) \geq eval(T, r', U_i)$.

Definition 5 (Connectedness on Association Rules of Changes) *The connectedness of condition and decision attributes for each relative reduct B of T is defined by*

$$connectedness(T, B) = \sum_{i=1,\ldots,|sign(V_d)|} max\_eval(T, R_B, U_i)$$

The underlying assumption behind the maximum evaluation $max\_eval(T, R_B, U_i)$ is that the connectedness is strengthened by the existence of one-sided association rules. That is, it is required that a rule has a high consistency evaluation in U_i rather than the total value of consistency evaluations for all the association rules in U_i. From the assumption, a maximum value of one is selected from the association rules of different conditions implying the same decision. For example, consider the above rule r_1 and the following rule r_3.

$$r_3\colon (a_1 = -1) \wedge (a_2 = +1) \rightarrow (d = +1)$$

A higher value of one is selected from the consistency evaluations $eval(T, r_1, U_1)$ and $eval(T, r_3, U_1)$ rather than their sum. This is because the two rules have opposite conditions $(a_1 = +1) \wedge (a_2 = -1)$ and $(a_1 = -1) \wedge (a_2 = +1)$ deriving the same decision $(d = +1)$. The sum of their consistency evaluations aggregates different behaviors, and therefore, it does not indicate the connectedness of condition and decision attributes.

We calculate the connectedness for each of the connected decision tables such that the association rules of changes are generated and evaluated by sliding the connection of two temporal information systems. From various time delays, a maximum connectedness is discovered as a strong connection of the systems.

4 Change and Connection Mining Algorithm

This section describes a change and connection mining algorithm for two temporal information systems T_1 and $T_2[b]$ and a maximum time delay

Algorithm: *Change and Connection Mining*

Input: temporal information systems T_1 and $T_2[b]$,
maximum time delay m $(\in Nat)$
Output: time delay t
1: **begin**
2: $T_1' = \pi_1(T_1[a_1]) \circ \cdots \circ \pi_n(T_1[a_n])$; $T_2' = \pi(T_2[b])$;
3: **for** $\Delta = 0$ **to** m **do**
4: $T = con(T_1', T_2', \Delta)$;
5: $\mathcal{R} = reducts(T)$; (based on I_B^{qc})
6: **for** $B \in \mathcal{R}$ **do**
7: $C_B = 0$;
8: **for** $i = 1$ **to** $|sign(V_d)|$ **do**
9: $max\_eval_i = 0$;
10: **for** $x \in U_i$ **do**
11: $r = rule(x, B, T)$;
12: $eval_i = eval(T, r, U_i)$; (using μ_S)
13: **if** $max\_eval_i < eval_i$ **then** $max\_eval_i = eval_i$;
14: **rof**
15: $C_B = C_B + max\_eval_i$;
16: **rof**
17: **rof**
18: $connectedness_\Delta = max(\{C_B \mid B \in \mathcal{R})$;
19: **rof**
20: **return** t $(connectedness_t = max(\{connectedness_j \mid 0 \le j \le m\}))$;
21: **end**;

Figure 4: The change and connection mining algorithm for two temporal information systems T_1 and $T_2[b]$.

$m(\in Nat)$. As shown in Figure 4, this algorithm generates and evaluates association rules of changes in the connections of T_1 and $T_2[b]$ for various time delays.

First of all, in order to analyze hidden behaviors in $T_1 = (U_{time}, \{a_1, \ldots, a_n\})$ and $T_2[b] = (U'_{time}, \{b\})$, quantitative estimation operators $\pi_1, \ldots, \pi_n$ and π are applied to $T_1[a_1], \ldots, T_1[a_n]$ and $T_2[b]$, and the estimated results are stored in the variables T_1' and T_2' (in Line 2). Since we are not certain which time delay constructs a temporal relation suitable for T_1' and T_2', this algorithm functions to connect them in varying time delays Δ from 0 to m (in Lines 3 - 19). In the loop of the time delays, the connected decision table $T = (U''_{time}, \{a_1, \ldots, a_n\} \cup \{b\})$ of T_1' and T_2' is computed by $T = con(T_1', T_2', \Delta)$ (in Line 4). Therefore, the connected decision table

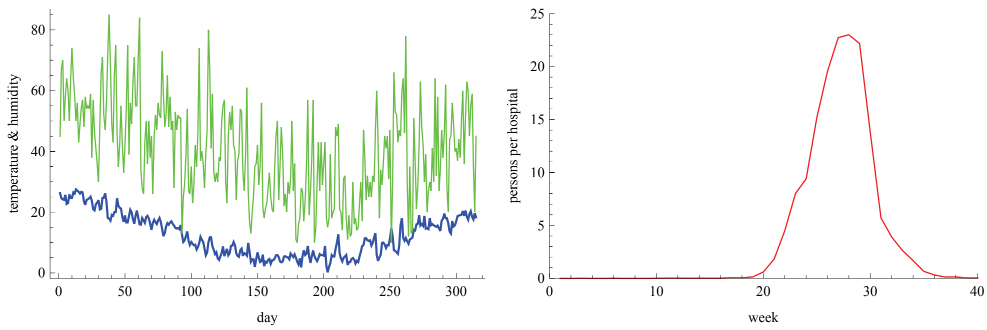

Figure 5: Climate data and medical data in Tokyo.

T is one of the candidate cause-effect decision tables to generate association rules of changes. Using the rough set theory, minimal association rules $r = rule(x, B, T)$ (as in Line 11) are created for each relative reduct B in $\mathcal{R} = reducts(T)$ [18], i.e., B is a subset of the set $\{a_1, \ldots, a_n\}$ in T that supplies a minimal set of condition attributes of rules. Computing the set $\mathcal{R} = reducts(T)$ of relative reducts of $T = con(T'_1, T'_2, \Delta)$ is based on the B-indiscernibility relation of quantitative changes by extending the standard reduct set computation in [4]. In other words, the reducts are computed by the standard reduct set algorithm that is extended by the B-indiscernibility relation of quantitative changes.

Let U_i be a decision class on quantitative changes where $U''_{time} = U_1 \cup \cdots \cup U_{|sign(V_b)|}$ in the connected decision table T. Each decision class on quantitative changes U_i corresponds to one of the values of $sign(V_b)$ of decision attribute b. Then, the association rules $r = rule(x, B, T)$ for all the time stamps x in U_i are generated for every $i = 1, \ldots, |sign(V_b)|$ (in Lines 8 - 16). The function $rule(x, B, T)$ simply determines a decision rule for each item x (in U) in information system T using each reduct of B. For each i from 1 to $|sign(V_b)|$, we obtain the maximum consistency evaluation (stored in the variable max_eval_i) by calculating $eval(T, r, U_i)$ (in Lines 12 - 13). Finally, the connectedness of T'_1 and T'_2 for each relative reduct B (stored in variable C_B) is calculated (in Line 15). For every current time delay Δ, we select the maximum connectedness denoted by $connectedness_\Delta$ from all the relative reducts B in $\mathcal{R}$ (in Line 18). After the loop of the time delays (Lines 3 - 19), a time delay t with the maximum connectedness is returned by comparing the connectedness for each time delay in $0 \leq j \leq m$.

5 Experimental Results

We implemented the change and connection mining algorithm in Java. In order to evaluate the proposed mining algorithm on real-world data, two

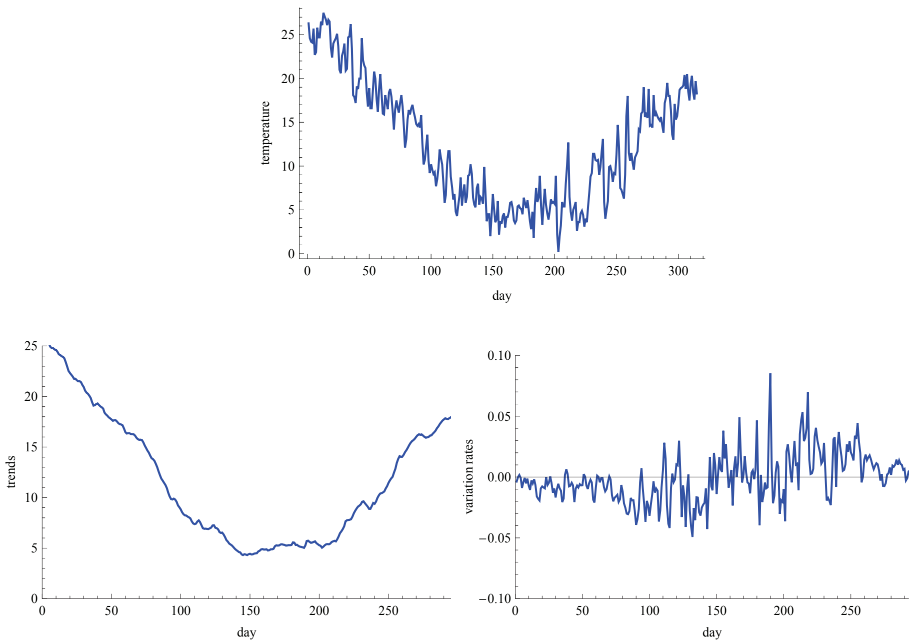

Figure 6: Temperature data in $T_1[a_t]$, and the quantitative changes $\pi_{tr(10)}(T_1[a_t])$ and $\pi_v(\pi_{tr(10)}(T_1[a_t]))$.

time-series data sets were built as shown in Figure 5, by downloading climate data and medical data in Tokyo from the web sites of [2, 1]. On the left-hand side of Figure 5, the climate data set contains the daily observed data of minimum temperature and humidity in Tokyo from August 7 2006 until July 15 2007. On the right-hand side of Figure 5, the medical data set consists of weekly reported numbers of influenza victims per hospital in Tokyo from September 4 2006 until June 17 2007. These data sets can be represented by the two normalized temporal information systems $T_1 = (N_{1,315}, \{temperature, humidity\})$ and $T_2 = (N_{5,45}, \{influenza\})$. The sizes of data sets and attributes are obtained from information systems, e.g., $T_1 = (N_{1,315}, \{temperature, humidity\})$ implies (daily) 315 items and two attributes. We simply denote the attributes *temperature*, *humidity*, and *influenza* by a_t, a_h, and b_f, respectively.

Let π_v, $\pi_{tr(10)}$, $\pi_{\leq 35}$, and π_d be quantitative estimation operators. Figures 6 - 8 demonstrate the results of applying the quantitative estimation operators to the climate data and the medical data of Tokyo.

For the climate data, first, to exclude noisy data the long-term behaviors of daily observed temperatures are estimated by the trends $\pi_{tr(10)}(T_1[a_t])$ of the temperatures in $T_1[a_t]$ on the left-hand side in Figure 6. Then, the increase-decrease rates of the trends are calculated by the variation rates $\pi_v(\pi_{tr(10)}(T_1[a_t]))$ from the data of the trends $\pi_{tr(10)}(T_1[a_t])$ on the right-

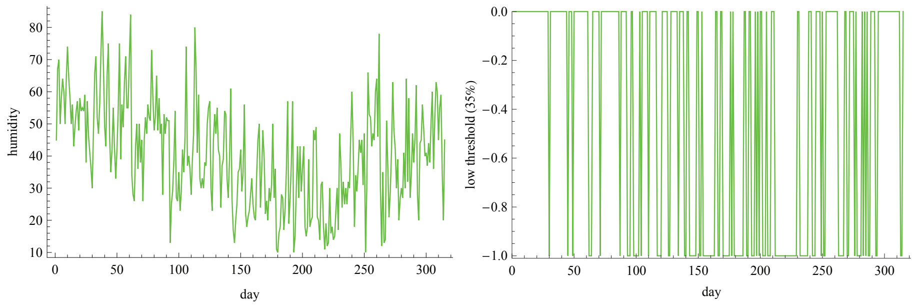

Figure 7: Humidity data in $T_1[a_h]$ and the quantitative change $\pi_{\leq 35}(T_1[a_h]))$.

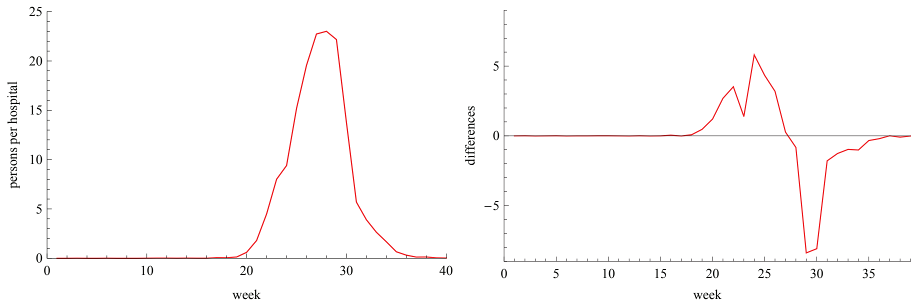

Figure 8: Influenza data in $T_1[b_f]$ and the quantitative change $\pi_d(T_2[b_f])$.

hand side. Second, we set the threshold such that the changes $\pi_{\leq 35}(T_1[a_h])$ of humidity in $T_1[a_h]$ are denoted by -1 if their values decrease to equal to or less than 35% at the time stamp. Figure 7 presents the daily observed humidity levels in the temporal information system $T_1[a_h]$ on the left-hand side. Then, the quantitative changes $\pi_{\leq 35}(T_1[a_h])$ are estimated from the data of $T_1[a_h]$ on the right-hand side.

For the medical data, the differences $\pi_d(T_2[b_f])$ of the numbers of influenza victims in $T_2[b_f]$ are regarded as candidate effects of the quantitative changes $\pi_v(\pi_{tr(10)}(T_1[a_t])) \circ \pi_{\leq 35}(T_1[a_h])$. Figure 8 shows the number of influenza victims for each week in the temporal information system $T_1[b_f]$ on the left-hand side and the differences $\pi_d(T_2[b_f])$ estimated from the data of $T_1[b_f]$ on the right-hand side. Unlike estimating the variation rates of temperatures, the numbers of victims are absolute values; therefore, the differences should be calculated (since absolute values contain 0).

After the estimation, we turn to connecting the climate data and the medical data by using the quantitative changes $\pi_v(\pi_{tr(10)}(T_1[a_t])) \circ \pi_{\leq 35}(T_1[a_h])$ and $exp_7(\pi_d(T_2[b_f]))$ of T_1 and $T_2[b_f]$ with an expansion function exp_7. Since $T_2[b_f]$ consists of weekly data, we expand it to daily data denoting the differences per week. Let $T = (Nat_{i,j}, A)$ and $k > 1$. Then, the expansion function is defined by $exp_k(T) = (U_{time}, A')$ such that

- $U_{time} = Nat_{(i-1)\cdot k+1, j\cdot k}$ and
- $A' = \{a' \mid a \in A \ \& \ \forall x \in U_{time}.a'(x) = a(quotient(x + k - 1, k))\}$.

Figure 9 represents the outcomes of applying the algorithm to the two temporal information systems T_1 and $T_2[b_f]$. Let T be the connected decision table $con(\pi_v(\pi_{tr(10)}(T_1[a_t])) \circ \pi_{\leq 35}(T_1[a_h]), exp_7(\pi_d(T_2[b_f])), \Delta)$. The left-hand side of Figure 9 shows the consistency evaluations $eval(T, r, U_i)$ for the time delays $\Delta = 0, \ldots, 13$ where r is an association rule $(a_t = a_t(x)) \wedge (a_h = a_h(x)) \rightarrow (b_f = b_f(x))$ for a time stamp $x \in U_i$ where $a_t(x) \in V_{a_t}$, $a_h(x) \in V_{a_h}$, and $b_f(x) \in V_{b_f}$. In order to overview the association rules with respect to the B-indiscernibility I_B^{qc}, each association rule is generalized by $(a_t = sign(a_t(x))) \wedge (a_h = sign(a_h(x))) \rightarrow (b_f = sign(b_f(x)))$ for a time stamp $x \in U_i$ where $sign(a_t(x)) \in \{-1, 0, +1\}$, $sign(a_h(x)) \in \{-1, 0\}$, and $sign(b_f(x)) \in \{-1, 0, +1\}$. In this figure above, the *generalized* association rules are listed by indexing the plot styles for their consistency evaluations. In particular, it can be seen that the generalized association rule $(a_t = -1) \wedge (a_h = -1) \rightarrow (b_f = +1)$ has a high consistency evaluation through the whole time delays, compared with the other rules. Note that the value of each generalized association rule in the figure totally indicates the evaluation obtained by measuring the weights of association rules with similar data behaviors. The high evaluation of the generalized rule reports that if the values of the temperature decrease and the humidity are equal to or under the 35 percent limit, then the number of influenza victims increases. Without the consistency evaluation and various time delays, it would be difficult to select a time-delayed decision table that consistently connects the different contexts of distributed data.

On the right-hand side of Figure 9, our experimental result indicates the connectedness evaluated for each of the time delays $\Delta = 0, \ldots, 13$. As indicated by the result in the figure, the algorithm returns the time delay $\Delta = 2$ (denoting two days) that has the maximum connectedness 0.2698 of T_1 and $T_2[b_f]$. Further, high connectedness values 0.2685 and 0.2617 are given for the time delays $\Delta = 3$ and 1 (denoting three days and one day). This result confirms the existence of certain time delays that strongly connect the climate data and the medical data of Tokyo.

6 Related Work

Many data mining algorithms for multiple databases have been proposed in the area of data mining and knowledge discovery. Cheung et al [5] developed a distributed algorithm DMA (for Distributed Mining of Association rules). This algorithm is efficient because the number of candidate itemsets is reduced by locally pruning itemsets in multiple databases. Jin and Agrawal [12] established new operators for querying frequent patterns across

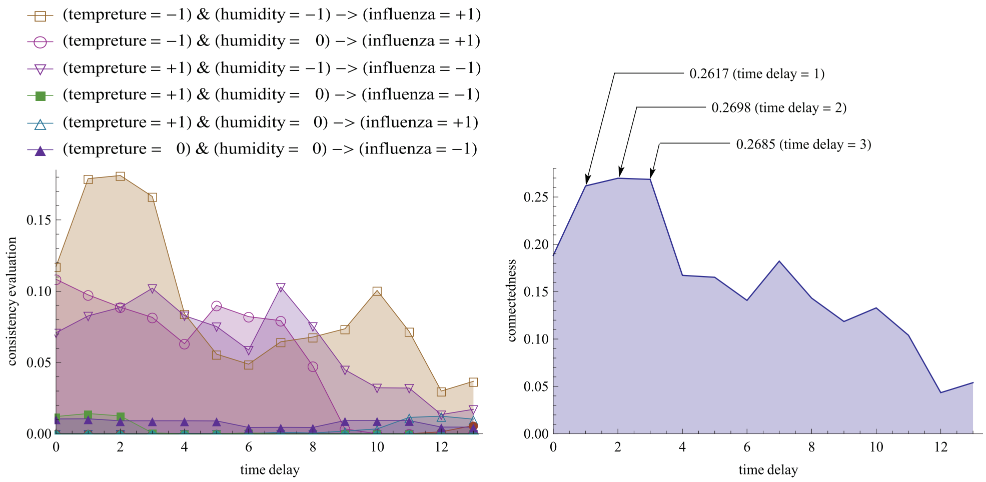

Figure 9: The consistency evaluation and the connectedness in $con(\pi_v(\pi_{tr(10)}(T_1[a_t])) \circ \pi_{\leq 35}(T_1[a_h]), exp_7(\pi_d(T_2[b_f]), \Delta))$.

multiple databases and presented an algorithm for finding an optimized query plan across multiple databases. These approaches can find frequent patterns in multiple databases; however they cannot analyze quantitative changes and temporal relationships in data across multiple databases. Zhu and Wu [25] developed an algorithm for discovering relational patterns where multiple databases are jointed together to construct a hybrid frequent pattern tree. The relational patterns are constraints described by relationships and operators on support values in multiple databases. Unlike their algorithm, our algorithm for mining changes and connections with some time delays is closely related to the mining of causality in time-series data, essentially differing from the frequent relational patterns.

As described in [14], several causal discovery systems have been developed such as CaMML, TETRAD, and TimeSleuth. Their algorithms and systems infer causal relationships based on Bayesian learning ideas. Silverstein et al.[23] proposed a constraint-based algorithm for discovering causality. They distinguished causality mining from traditional association rule mining as simply finding a statistical relationship between itemsets. Our method is related to such causal discovery research because the minimal and consistent rules with time delays follow some notions of the causal theories. In comparison, their methods basically analyze the values of original data; however, our work attempts to estimate quantitative changes (including fast changing events) and time-delayed connections in the data of two temporal databases. In other words, our algorithm formulates association rules under the indiscernibility of unifying similar data behaviors and the weight of measuring quantities. These rules are significantly used to evaluate the connectedness of two databases for each of different time delays.

In the area of the rough set theory, several studies have been undertaken on mining temporal data. Hirano and Tsumoto [8] presented a method for finding patterns from spatio-temporal data using rough set based clustering. This approach can group sequences from a single spatio-temporal information system wherein data are associated with time and spatial positions. In comparison, our work proposes to distinguish between the indiscernibility and weight in the rule generation so that fast changing events and time-delayed connections can be detected from the quantitative changes in distributed data.

As a rough set research, Milton , Maheswari, and Siromoney [17] proposed to simply combine multiple information systems on the axis of common attributes. Their work is based on the relational learning of multiple relations in inductive logic programming. However, in the research, no evaluation criteria for connecting multiple information systems were presented and no algorithm for analyzing quantitative changes and time delays between two information systems was considered. In addition, Inuiguchi [9] extended a rule induction method for multi-agent rough sets. His method can analyze a conflict tolerance of multiple decision tables that have identical objects and attributes. In contrary, we consider multiple information systems with different attributes for analyzing their changes and time-delayed connections. By using the time stamps of data, a connected decision table is constructed such that one information system is used as condition attributes and the other information system is used as decision attributes.

7 Conclusion and Future Work

We have proposed a method for mining the changes and time-delayed connections from two temporal information systems in the rough set theory. As a novel approach, we establish a distinction between the indiscernibility and weight of quantitative changes in the rough-set rule generation and evaluation. Our method contains the quantitative estimations for extracting the changes in numeric values from different data sets. The proposed mining algorithm slidingly connects the quantitative changes in distributed data to generate (candidate) cause-effect decision tables for various time delays. We devise an evaluation method for the consistency in the association rules of changes by adjusting weight-based accuracy and coverage in order to compute the connectedness between two information systems. The experimental result indicates that our method can discover certain time delays that causally connect the climate data and the medical data of Tokyo.

Our future research is concerned with establishing an algebra for complex queries of the quantitative estimation operators in order to discover changes and connections across multiple information systems for *users' requirements.*

In practical applications, users could indicate such complex queries that help decide the direction to capture the implicit essence of data when data mining relies on domain knowledge.

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